Apostol M, J Phys Res Appl 2024, 8:4

Journal of Physics Research and Applications

Short Communication

A SCITECHNOL JOURNAL

A Note on the Solar Neutrino Problem

M. Apostol[*]

Department of Theoretical Physics, Institute of Atomic Physics, Magurele-Bucharest Mg-6, POBox MG-35, Romania

[*]**Corresponding author:** M. Apostol, Department of Theoretical Physics, Institute of Atomic Physics, Magurele-Bucharest Mg-6, POBox MG-35, Romania; E-mail: apoma@theory.nipne.ro

Received date: 07 November, 2023, Manuscript No. JPRA-23-119493;

Editor assigned date: 09 November, 2023, PreQC No. JPRA-23-119493 (PQ);

Reviewed date: 22 November, 2023, QC No. JPRA-23-119493;

Revised date: 06 December, 2024, Manuscript No. JPRA-23-119493 (R);

Published date: 13 December, 2024, DOI: 10.4172/jpra.1000122

Abstract

The coupled Dirac equations for solar e- and μ-neutrinos are solved in the presence of a small uniform interaction of the e-neutrinos with matter (the Sun). It is shown that the interaction leads to an increase of the amplitude of the neutrinos oscillations and a decrease of the localization probability of the e-neutrinos. However, the strength of this interaction is insufficient to explain the magnitude of the discrepancy between the solar neutrino flux and the neutrino flux measured on the Earth. An alternative interpretation related to a possible superfluid neutrino state is advanced, which may throw some light on the solar neutrino problem.

Keywords: Solar neutrinos; Neutrino oscillations; Coupled Dirac equations; Interaction with matter; Admixture angle; Superfluid correlations

Introduction

It is well known that the solar e-neutrinos measured on the Earth are less than the value expected from models of neutrino production in the Sun [1,2]. In order to solve this solar neutrino problem an interaction of the e-neutrinos with matter was suggested [3,4]. It is claimed that such an interaction produces a difference in mass, which would favour the μ-neutrinos [5-9]. However, mass is a Casimir invariant of the Lorentz group which is not affected by interaction. The interaction affects momentum and energy. The coupled Dirac equations of the e- and μ-neutrinos are solved here for a small uniform interaction. It is shown that interaction leads to an increase in the oscillation amplitude of the e-electrons and a decrease of their localization probability. This effect may be related to the solar neutrino problem, though it is much weaker than what would be needed to explain the discrepancy.

Description

Oscillations

We consider two neutrino eigenstates of the free hamiltonian $\upsilon\alpha$, α=1, 2, corresponding to admixtures of e- and μ-neutrinos; with usual notations their wave functions (spinors) are

$$\nu_\alpha(\mathbf{r},t)=e^{-iE_\alpha t+i\mathbf{pr}}\nu_\alpha,\ \text{where}\ E_\alpha=\sqrt{m_\alpha^2+p^2}$$

is energy, p is momentum and $m\alpha$ is the mass. The wave functions of the e- and μ-neutrinos are [10-14]

$$\begin{gathered}\psi_e=\cos\theta\cdot\nu_1+\sin\theta\cdot\nu_2\quad,\quad\psi_\mu=-\sin\theta\cdot\nu_1+\cos\theta\cdot\nu_2\ ,\\ \nu_1=\cos\theta\cdot\psi_e-\sin\theta\cdot\psi_\mu\quad,\quad\nu_2=\sin\theta\cdot\psi_e+\cos\theta\cdot\psi_\mu\ ,\end{gathered}\tag{1}$$

where θ is the admixture angle [15]. We write also

$$\psi=U\nu\ ,\ \nu=U^{-1}\psi\ ,\tag{2}$$

where $\psi=(\psi_e,\psi_\mu)^T$, $\nu=(\nu_1,\nu_2)^T$ and the matrix U is

$$U=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\ .\tag{3}$$

The wavefunction $\psi_e(\mathbf{r},t)$ can be written as

$$\begin{gathered}\psi_e(\mathbf{r},t)=\cos\theta e^{-iE_1t+i\mathbf{pr}}\nu_1+\sin\theta e^{-iE_2t+i\mathbf{pr}}\nu_2=\\ =\left[\cos^2\theta e^{-iE_1t+i\mathbf{pr}}+\sin^2\theta e^{-iE_2t+i\mathbf{pr}}\right]\psi_e-\\ -\cos\theta\sin\theta\left(e^{-iE_1t+i\mathbf{pr}}-e^{-iE_2t+i\mathbf{pr}}\right)\psi_\mu\ ,\end{gathered}\tag{4}$$

hence the probability of localization

$$\begin{gathered}|\psi_e(\mathbf{r},t)|^2=\left(1-\sin^2 2\theta\sin^2\frac{\Delta Et}{2}\right)|\psi_e|^2\ ,\\ |\psi_\mu(\mathbf{r},t)|^2=\sin^2 2\theta\sin^2\frac{\Delta Et}{2}|\psi_\mu|^2\ ,\end{gathered}\tag{5}$$

where $\Delta E=E_2-E_1$; we can see the oscillations e ↔ μ. Similar oscillations occur for $E_1=E_2$ and distinct p's.

Such neutrinos are produced in the Sun with energies of the order a few MeV 's; as it is well known, their mass is very small (or even zero). For instance, in the Sun the e-neutrinos interact with electrons; for 1 MeV the neutrino wavelength is $\simeq 2\times10^{-11}$ cm, which is comparable to the electron Compton wavelength. It follows that the interaction is mainly a forward scattering. The strength of the interaction of a neutrino with an electron is of the order G/Ω, where G $\simeq 10^{-49}$erg $\cdot$cm^3 is the weak-interaction coupling constant and Ω is the volume; it follows that the interaction of an e-neutrino with the medium can be written as

$$V=\frac{G}{N}\sum_i\delta(\mathbf{r}-\mathbf{r}_i)\ ,\tag{6}$$

where N is the number of scattering centers, labelled by i; in a first approximation it can be taken as V=Gn $\simeq 10^{-24}$erg ($\simeq 10^{-12}$eV), where n is the density of electrons (in the Sun n $\simeq 10^{25}$cm^{-3}). This approximation of a uniform interaction corresponds to neutrinos with relatively moderate energy; for higher energies than the order of localization energy in the nucleus (≃ 10MeV) we may expect anomalies in the interaction effects. Equations (1) and the Dirac equation

$$i\frac{\partial\nu_\alpha}{\partial t}=(\boldsymbol{\alpha}\mathbf{p}+\beta m_\alpha)\nu_\alpha \tag{7}$$

lead to the Dirac equations

$$\begin{aligned} i\tfrac{\partial\psi_e}{\partial t}&=[\boldsymbol{\alpha}\mathbf{p}+(m-\Delta m\cos 2\theta)\beta]\,\psi_e+\Delta m\sin 2\theta\cdot\beta\psi_\mu\ ,\\ i\tfrac{\partial\psi_\mu}{\partial t}&=[\boldsymbol{\alpha}\mathbf{p}+(m+\Delta m\cos 2\theta)\beta]\,\psi_\mu+\Delta m\sin 2\theta\cdot\beta\psi_e \end{aligned} \tag{8}$$

For the e, μ-neutrinos (α and β are the Dirac matrices), where $2m=m_1+m_2$ and $2\Delta m=m_2-m_1$; their solutions are given by equations (1). These equations can also be written as

$$i\frac{\partial\psi}{\partial t}=U\overline{E}\nu\ ,\ \ \overline{E}=\begin{pmatrix}E_1 & 0\\ 0 & E_2\end{pmatrix}\ . \tag{9}$$

Interaction

In the presence of the interaction of the e-neutrinos with matter, these equations become

$$i\frac{\partial\widetilde{\psi}}{\partial t}=U\overline{E}\widetilde{\nu}+\overline{V}\widetilde{\psi}\ , \tag{10}$$

Where

$$\overline{V}=V\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\ , \tag{11}$$

as

$$i\frac{\partial\widetilde{\nu}}{\partial t}=\overline{E}\widetilde{\nu}+U^{-1}\overline{V}U\widetilde{\nu}\ . \tag{12}$$

We seek the solution as

$$\widetilde{\nu}=e^{-iEt}\mu\ , \tag{13}$$

where μ satisfies the system of equations

$$\begin{aligned} i\tfrac{\partial\mu_1}{\partial t}&=V\cos^2\theta\cdot\mu_1+V\cos\theta\sin\theta e^{-i\Delta Et}\mu_2\ ,\\ i\tfrac{\partial\mu_2}{\partial t}&=V\cos\theta\sin\theta e^{i\Delta Et}\mu_1+V\sin^2\theta\cdot\mu_2\ . \end{aligned} \tag{14}$$

We solve this system of equations in the limit V ≪ ΔE; the solution is

$$\begin{aligned} \widetilde{\nu}_1&\simeq e^{-i(E_1-V\cos^2\theta)t}\nu_1-\tfrac{V}{2\Delta E}\sin 2\theta\left(e^{-iE_1t}-e^{-iE_2t}\right)\nu_2\ ,\\ \widetilde{\nu}_2&\simeq e^{-i(E_2-V\sin^2\theta)t}\nu_2-\tfrac{V}{2\Delta E}\sin 2\theta\left(e^{-iE_1t}-e^{-iE_2t}\right)\nu_1\ . \end{aligned} \tag{15}$$

The probability of localization of the e-neutrinos, interaction included, is

$$\begin{aligned} \left|\widetilde{\psi}_e(\mathbf{r},t)\right|^2&=\{1-\sin^2 2\theta[\sin^2\tfrac{(\Delta E+V\cos 2\theta)t}{2}+\\ &\quad+\tfrac{2V}{\Delta E}\sin^2\tfrac{\Delta Et}{2}]\}\left|\widetilde{\psi}_e\right|^2\ . \end{aligned} \tag{16}$$

The time average of this probability shows that the interaction leads to an increase of the oscillations amplitude (decrease of the localization probability) by a factor 1+2V/ΔE (ΔE >0). It is worth noting that for ΔE=0 (equal, or vanishing mass) the oscillations are given by $1-\sin^2 2\theta\ \sin^2(V\ t\cos 2\theta/2)$. Similar calculations can be done for an admixture of three neutrino flavours, to include the τ-neutrino, with similar results.

Conclusion

In conclusion, a small uniform interaction between e-neutrinos and matter is considered here. The coupled Dirac equations of the e, μ-neutrinos are solved for this interaction and the neutrino oscillations are computed. It is shown that the oscillation amplitude of the e-neutrinos decreases in the presence of the interaction and their localization probability increases. Although the effect may be related to the solar neutrino problem, it is much weaker (V/ΔE ≪ 1) than what would be needed to explain the magnitude of the solar neutrino problem. It is likely that the solar neutrinos have a very small temperature, as a result of their extremely weak interaction with matter. Consequently, they may be viewed as being in their ground state, which is a Fermi sea. Since the electron and neutrino Compton wavelengths are close to each other we may expect an attractive neutrino-neutrino interaction from their forward scattering. This interaction may be mediated by the longitudinal compression solar density waves. Consequently, a superfluid ("superconducting") neutrino-pairing state may be expected. The combination of the weak interaction strength and the state density may lead to a critical temperature which lies above the neutrino temperature, and a corresponding energy gap. Therefore, a sizeable depletion of neutrino states might be expected to paired states. A paired state, extended in the whole space, can be viewed as a composite particle with internal cohesion energy; such a state would not be detectable, which may give a hint to the solar neutrino problem.

Acknowledgment

The author is indebted to his friends S. Stoica for bringing this problem to his attention and M. Horoi for useful comments. Also, the author is indebted to the members of the Laboratory of Theoretical Physics at Magurele-Bucharest for many fruitful discussions. This work has been supported by the Scientific Research Agency of the Romanian Government through Grants 04-ELI/2016 (Program 5/5.1/ELI-RO), PN 16 4201 01/2016 and PN (ELI) 16 42 01 05/2016

Conflict of Interest

The author declares no conflict of interest.

References

1. Greiner W, Muller B (1996) Gauge theory of weak interactions. New York: Springer.
2. Kumaran S, Ludhova L, Penek O, Settanta G (2021) Borexino results on neutrinos from the Sun and Earth. Universe 7: 231.

3. Wolfenstein L (2018) Neutrino oscillations in matter. InSolar neutrinos. CRC Press.
4. Wolfenstein L (1979) Neutrino oscillations and stellar collapse. Phys Rev D 20: 2634.
5. SP. Mikheyev, AYu. Smirnov (1985) Resonant amplification of neutrino oscillations in matter and spectroscopy of solar neutrinos. Sov J Nucl Phys 42: 913-917.
6. Mikheev SP, Smirnov AY (1986) Neutrino oscillations in a variable density medium and neutrino bursts due to the gravitational collapse of stars. Sov Phys JETP
7. Mikheyev SP, Smirnov AY (1989) Resonant neutrino oscillations in matter. Prog Par Nuc Phys 23: 41-136.
8. Bethe HA (2018) Possible explanation of the solar-neutrino puzzle. Phys Rev Lett 56: 1305-1308
9. Smirnov AY (2016) Solar neutrinos: Oscillations or No-oscillations?.
10. Mannheim PD (1988) Derivation of the formalism for neutrino matter oscillations from the neutrino relativistic field equations. Phys Rev D Part Fields 37: 1935.
11. Giunti C, Kim CW, Lee UW, Lam WP (1992) Majoron decay of neutrinos in matter. Phys Rev D Part Fields 45: 1557.
12. Giunti C, Kim CW, Lee UW (1992) Remarks on the weak states of neutrinos. Phys Rev D Part Fields 45: 2414.
13. Blasone M, Henning PA, Vitiello G (1999) The exact formula for neutrino oscillations. Phys Lett B 451: 140-145.
14. Horoi M (2020) On the MSW-like neutrino mixing effects in atomic weak interactions and double beta decays. Europ Physl J A 56: 1-10.
15. Gonzalez-Garcia MC, Nir Y (2003) Neutrino masses and mixing: Evidence and implications. Rev Modern Phys 75: 345.